

Section Handout 2

Problem One: Geometric Series

A **geometric series** is a series $ar^0, ar^1, ar^2, ar^3, \dots, ar^{n-1}$ where a and r are real numbers. Prove, by induction on n , that for $r \neq 1$,

$$\sum_{i=0}^{n-1} ar^i = a \frac{1-r^n}{1-r}$$

Problem Two: Picking Coins

Consider the following game for two players. We begin with a pile of n coins for some $n \geq 0$. The first player then takes between one and ten coins out of the pile, then the second player takes between one and ten coins out of the pile. This process repeats until some player has no coins to take; at this point, that player loses the game. Prove that if the pile begins with a multiple of eleven coins in it, the second player can always win.

Problem Three: Prime Numbers

A natural number $p \geq 2$ is called **prime** if it has no positive divisors except 1 and itself. A natural number is called **composite** if it is the product of two natural numbers m and n , where both m and n are greater than one. Prove, by strong induction, that every natural number greater than one can be written as a product of prime numbers.

Problem Four: Induction on Integers

In lecture, we discussed how to use induction to prove properties of natural numbers. However, induction can also be used to prove properties of integers as well, so long as a suitably modified form of induction is used.

Let $P(x)$ be a property that we wish to show is true of all integers and suppose that we can show the following:

- $P(0)$.
- For any integer x , $P(x) \rightarrow P(x + 1)$
- For any integer x , $P(x) \rightarrow P(x - 1)$

Prove, using the definition of the principle of mathematical induction, that $P(x)$ holds for all integers x . (*Hint: Induction is defined to only work over the natural numbers. Can you think of a property $Q(n)$ that you could show is true for all natural numbers such that $Q(n)$ being true for all natural numbers implies that $P(x)$ is true for all integers?*)